Two-Phase Load Modelling in Three-Phase Load Flow

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Currents and voltages of the power system are not only dependent on the elements of the power system, but also on the loads that are being connected onto the system. The aim of this article is to present two-phase load modelling and its implementation to the Newton-Raphson method for calculation of the three-phase load flow. The program, based on the method, had been made and was tested in the Croatian power system where one of the main causes of the asymmetry is the Electrical Traction System with a two-phase connection onto the power system through the transformer 110/25 kV.

Keywords: two-phase load modelling, Newton-Raphson method, three-phase load flow, electrical traction system

1. Introduction

In the calculation of the single-phase load flows we are starting from the stand point of symmetrical conditions in the power system. In order to achieve greater precision in the analysis of the asymmetries in the power system, it is necessary to introduce the three-phase modelling to all elements of the system. The loads present a special concern and have to be modelled, too. In Croatia, a major problem occurs when the Electrical Traction System (1×25 kV) is being connected onto the power system - Traction Substations are mounted onto the power system through the single-phase transformers between two phases of the 110 kV network.

Decoupled Newton-Raphson method [1, 2] is used for the calculation of the three-phase load flow. The lines have a three-phase modelling and are represented by the 3×3 matrices. In the case of parallel lines, the influence that one line has on the other is also considered. When modelling a transformer, it is important to know the type of the connection and the turn ratio. Generator model is created with the help of the positive, negative and zero sequence reactance, having in mind the star grounding. Unlike standard methods for the three-phase load flow calculation, this article emphasises the two-phase load modelling. In the calculation, the loads are presented in the form of constant two-phase power. This comes from the Traction Vehicle characteristics. Therefore, it is necessary to make adjustments to the standard method by including the two-phase modelling.

2. Power System Modelling for Three Phase Load Flow

2.1. Transmission Line, Transformer and Generator Models

Single-phase modelling of the elements in the power system is used for the calculation of the three-phase load flow in symmetrical conditions. Three-phase modelling gives much clearer and better picture of the current and voltage relations and conditions in the network because it takes into consideration the following factors:

a) long untransposed lines

b) mutual influence of the parallel lines

c) current and voltage shift due to the different type of transformer connection

d) different types of transformer and generator star grounding and

e) other unbalances of the power system elements.

In case of single-phase models, all the power system elements are modelled by the series impedance and shunt susceptance. Three-phase models are represented by the 3×3 matrices.

Transmission lines

In line modelling, Carson's formulas are used to find the self- and mutual-impedances and potential coefficients for self- and mutual-capacitances [3, 4]. Basic matrix equation for the current and voltage can be found based on the existing series and shunt admittance matrix.

$$\begin{bmatrix} [I_i^{abc}] \\ [I_j^{abc}] \end{bmatrix} = \begin{bmatrix} [Y_u] + \frac{[Y_p]}{2} & -[Y_u] \\ -[Y_u] & [Y_u] + \frac{[Y_p]}{2} \end{bmatrix} \cdot \begin{bmatrix} [V_i^{abc}] \\ [V_j^{abc}] \end{bmatrix} \tag{1}$$

where $[Y_u]$ and $[Y_p]$ represent the series admittance matrix and the shunt admittance matrix, respectively.

In the case of parallel lines with mutual electromagnetic coupling, connected onto the nodes i, j and k, l, the final matrix equation has a more complex form:

$$\begin{bmatrix} [I_i^{abc}] \\ [I_j^{abc}] \\ [I_k^{a'b'c'}] \\ [I_l^{a'b'c'}] \end{bmatrix} = [\mathrm{Y}_{\mathrm{i,j-k,l}}] \cdot \begin{bmatrix} [V_i^{abc}] \\ [V_j^{abc}] \\ [V_k^{a'b'c'}] \\ [V_l^{a'b'c'}] \end{bmatrix} \tag{2}$$

where the matrix $[Y_{i,j-k,l}]$ is given by:

$$\begin{bmatrix} [Y_u]_1+[Y_p']_1 & -[Y_u]_1 & [Y_u]_{1-2}+[Y_p']_{1-2} & -[Y_u]_{1-2} \\ -[Y_u]_1 & [Y_u]_1+[Y_p']_1 & -[Y_u]_{1-2} & [Y_u]_{1-2}+[Y_p']_{1-2} \\ [Y_u]_{2-1}+[Y_p']_{2-1} & -[Y_u]_{2-1} & [Y_u]_2+[Y_p']_2 & -[Y_u]_2 \\ -[Y_u]_{2-1} & [Y_u]_{2-1}+[Y_p']_{2-1} & -[Y_u]_2 & [Y_u]_2+[Y_p']_2 \end{bmatrix} \tag{3}$$

a, b, c and a', b', c' represent phase conductors between the node i, j and k, l. Submatrices with indexes 1 and 2 represent the series impedance and shunt susceptance values of the line 1 (between the i, j node), and the line 2 (between the k, l node). Submatrices with indexes $1-2$ and $2-1$ are created by the line's mutual coupling.

Transformers

In the process of the transformer three-phase modelling, the mutual influence of the transformer windings can be considered sufficiently irrelevant and was therefore not considered. That allows us to substitute three-phase transformer modelling with the combination of three single-phase transformers, having in mind the type of the transformer connection [5]. The following matrix equation represents a transformer of the Yy-0 type with a grounded star:

$$\begin{bmatrix} I^A \\ I^B \\ I^C \\ I^a \\ I^b \\ I^c \end{bmatrix} = \begin{bmatrix} y_\alpha & 0 & 0 & -y_{\alpha\beta} & 0 & 0 \\ 0 & y_\alpha & 0 & 0 & -y_{\alpha\beta} & 0 \\ 0 & 0 & y_\alpha & 0 & 0 & -y_{\alpha\beta} \\ -y_{\alpha\beta} & 0 & 0 & y_\beta & 0 & 0 \\ 0 & -y_{\alpha\beta} & 0 & 0 & y_\beta & 0 \\ 0 & 0 & -y_{\alpha\beta} & 0 & 0 & y_\beta \end{bmatrix} \cdot \begin{bmatrix} V^A \\ V^B \\ V^C \\ V^a \\ V^b \\ V^c \end{bmatrix} \tag{4}$$

A, B, C - transformer's high voltage side phases

a, b, c - transformer's low voltage side phases

$y_\alpha = \frac{y_T}{\alpha^2}$ - y_T is leakage transformer admittance

- α is high voltage side turn ratio

$y_\beta = \frac{y_T}{\beta^2}$ - β is low voltage side turn ratio .

$y_{\alpha\beta} = \frac{y_T}{\alpha\beta}$.

For the transformer with the Yd-5 type of connection with direct star grounding (the one often found in the transmission networks) the following matrix equation is given: (5)

$$\begin{bmatrix} I^A \\ I^B \\ I^C \\ I^a \\ I^b \\ I^c \end{bmatrix} = \begin{bmatrix} y_\alpha & 0 & 0 & y_{\alpha\beta} & 0 & -y_{\alpha\beta} \\ 0 & y_\alpha & 0 & -y_{\alpha\beta} & y_{\alpha\beta} & 0 \\ 0 & 0 & y_\alpha & 0 & -y_{\alpha\beta} & y_{\alpha\beta} \\ y_{\alpha\beta} & -y_{\alpha\beta} & 0 & 2y_\beta & -y_\beta & -y_\beta \\ 0 & -y_{\alpha\beta} & -y_{\alpha\beta} & -y_\beta & 2y_\beta & -y_\beta \\ -y_{\alpha\beta} & 0 & y_{\alpha\beta} & -y_\beta & -y_\beta & 2y_\beta \end{bmatrix} \cdot \begin{bmatrix} V^A \\ V^B \\ V^C \\ V^a \\ V^b \\ V^c \end{bmatrix} \tag{5}$$

Similar procedure is applied when dealing with the transformer and the same type of connection, but with the grounding of the one or both stars through the impedance or in the case of isolated stars. The matrix order increases for one or two, i.e., the matrix becomes of the order 7×7 or 8×8. Injected power in these additional nodes is 0, so the nodes can be eliminated by the block transformation method. The result is a 6×6 matrix.

Generators

The generator is modelled by the symmetrical three-phase voltage source connected between

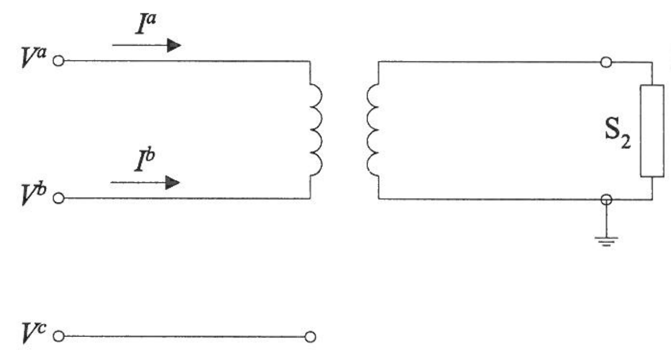

Fig. 1. Traction Substation Connection onto the Power System.

the star and internal busbars. Generator winding's self and mutual impedances values are related with the following matrix equation:

$$[Z^{abc}]_{GEN} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} Z_d & 0 & 0 \\ 0 & Z_i & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad (6)$$

$$= \frac{1}{3} \begin{bmatrix} Z_0 + Z_d + Z_i & Z_0 + aZ_d + a^2 Z_i & Z_0 + a^2 Z_d + aZ_i \\ Z_0 + a^2 Z_d + aZ_i & Z_0 + Z_d + Z_i & Z_0 + aZ_d + a^2 Z_i \\ Z_0 + aZ_d + a^2 Z_i & Z_0 + a^2 Z_d + aZ_i & Z_0 + Z_d + Z_i \end{bmatrix}$$

where $a = e^{j120^\circ} = -0.5 + j0.866$
$a^2 = e^{j240^\circ} = -0.5 - j0.866$
Z_d = positive sequence impedance
Z_i = negative sequence impedance
Z_0 = zero sequence impedance.

Generator's star grounding impedance is to be added to the zero sequence impedance. The value of this impedance is 0 in the case of the direct grounding, while in the case of isolated star it becomes infinite.

2.2. Load Modelling

In the real power system unbalanced loads (traction motors, induction furnaces, etc.) are commonly found as well as symmetrical ones. The biggest source of the asymmetry in the Croatian power system are the Electrical Traction Substations. The loads in the Electrical Traction System (Trains with thyristor and diode-controlled AC locomotive) are characterised by constant power. Due to the connection type of the traction substations onto the power system (Figure 1) the loads in the power system have to be modelled by the constant two-phase power.

If the power losses in the single-phase transformer are considered irrelevant, then the two-phase power S_2 in the i-th node is:

$$S_{i2} = (V_i^a - V_i^b) \cdot I_i^{a*}. \quad (7)$$

From the same equation, the current I can be defined as:

$$I_i^{a*} = \frac{S_{i2}}{V_i^a - V_i^b} \quad (8)$$

We can obtain the equations for the per phase powers by using the fact that $I^a = -I^b$:

$$S_i^a = V_i^a \cdot I_i^{a*} = V_i^a \cdot \frac{S_{i2}}{V_i^a - V_i^b} \quad (9)$$

$$S_i^b = V_i^b \cdot I_i^{b*} = -V_i^b \cdot \frac{S_{i2}}{V_i^a - V_i^b} \quad (10)$$

$$S_i^c = 0. \quad (11)$$

3. Newton-Raphson Method for Solving Three-Phase Load Flow Problem

In the calculation for the three-phase load flow, the following types of nodes exist:

- load busbar (P, Q)
- generator busbar (P, V)
- swing bus (V, δ)

For load busbar the known values are the constant load powers connected onto their busbars. The value and the angle of the phase voltages are, however, unknown and have to be found for this type of node.

Generator busbar (P, V) is modelled for the load flow calculation with the help of generator's terminal and internal busbars. Terminal busbars are the real busbars in the network where the generator is connected, while the internal busbars are fictive and represent the point between the symmetrical voltage source and the generator positive sequence reactance with the representative equations:

$$V_i^a = V_i^b = V_i^c = E_i \quad (12)$$

$$\delta_i^a = 120^\circ + \delta_i^b = -120^\circ + \delta_i^c = \delta_i \quad (13)$$

where V_i^a, V_i^b, V_i^c are the voltage magnitudes and δ_i^a, δ_i^b, δ_i^c are their angles.

Synchronous generator's excitement regulation keeps the terminal busbar voltage at the constant level. Active power which the generator nodes put into the network is given on its internal busbars and is equal to the per phase active power sum.

Swing bus is modelled almost the same way as the generator node. This type of node, on its terminal busbars, is given by the voltage magnitude and angle on the internal busbars.

It is necessary to add all the internal (fictive) nodes of all the generators in the network to the existing number of three-phase power system nodes. If the number of all real nodes in the network (made out of (P, Q) nodes, terminal busbars (P, V) and reference node terminal busbars) equals n_{bus}, and the number of all the internal busbars of the (P, V) nodes and the reference node equals n_{gen}, then the total number of nodes equals: $n = n_{bus} + n_{gen}$.

For all the (P, Q), terminal (P, V) nodes and the terminal busbars there is one basic power equation:

$$S_i^p = V_i^p \sum_{k=1}^{n-1} \sum_{q=a}^{c} (Y_{ik}^{pq} \cdot V_k^q)^* = V_i^p \sum_{k=1}^{n-1} \sum_{q=a}^{c} (G_{ik}^{pq} - jB_{ik}^{pq})(V_k^q)^* \quad (14)$$

where:

- n - total number of nodes
- S_i^p - i-th node power at phase p
- V_i^p - i-th node complex voltage magnitude at phase p
- $Y_{ik}^{pq} = G_{ik}^{pq} + jB_{ik}^{pq}$ - system admittance matrix element of the three-phase system

Separation of the active and reactive power in the equation (3.3) yields the following:

$$P_i^p = V_i^p \sum_{k=1}^{n-1} \sum_{q=a}^{c} V_k^q \cdot (G_{ik}^{pq} \cos(\delta_{ik}^{pq}) + B_{ik}^{pq} \sin(\delta_{ik}^{pq})) \quad (15)$$

$$Q_i^p = V_i^p \sum_{k=1}^{n-1} \sum_{q=a}^{c} V_k^q \cdot (G_{ik}^{pq} \sin(\delta_{ik}^{pq}) - B_{ik}^{pq} \cos(\delta_{ik}^{pq})) \quad (16)$$

To find the total active power, for the internal busbars (P, V) the generator puts into the network we have to add all the per phase active powers. The same mathematic procedure as the one for the previous node gives us:

$$Pgen_j = \sum_{p=a}^{c} \mathrm{Vint}_j \sum_{k=1}^{n-1} \sum_{q=a}^{c} V_k^q \cdot (G_{ik}^{pq} \cos(\delta_{ik}^{pq}) + B_{ik}^{pq} \sin(\delta_{ik}^{pq})) \quad (17)$$

where $j = (n_{bus} + 1), f \ldots, (n_{bus} + n_{gen} - 1)$, and V_{int} is internal busbars voltage of the (P, V) node.

For the swing bus and all the generator busbars some of the voltages of the terminal busbars have to be kept at the constant level. Often it is a voltage of one phase, so the equation looks like this:

$$Vreg_j = Vterm_j^a \quad (18)$$

where $j = (n_{bus} + 1), \ldots, n_{bus} + n_{gen}$.

Equations presented in the forms (15)-(18) are the basis for the three-phase load flow problem solving by the Newton-Raphson method. The following linear equation system is derived from the equations mentioned above:

$$\begin{aligned} \Delta P_i^p &= (P_i^p)^{SCHEDULED} - (P_i^p)^{CALCULATED} \\ \Delta Q_i^p &= (Q_i^p)^{SCHEDULED} - (Q_i^p)^{CALCULATED} \\ \Delta Pgen &= (Pgen_j)^{SCHEDULED} - (Pgen_j)^{CALCULATED} \\ \Delta Vreg_j &= (Vterm_j)^{SCHEDULED} - (Vterm_j^a)^{CALCULATED} \end{aligned} \quad (19)$$

The values that have to be determined by solving the equation system (19) are:

- voltages (magnitudes and angles) of all the load nodes and generator terminal busbars (including the reference node),
- voltage magnitudes of all the generator internal busbars (incl. reference)

- voltage angles of all the generator internal busbars (except reference, its angle is supposed to be 0).

While solving the equation system (19), the increase in the voltage magnitude which depends upon the active power increase is neglected, as well as the increase in the voltage angles which depend upon the reactive power increase. The voltage is kept at the constant level only by its magnitude and not by its angles, so their mutual dependence can be neglected, too. This allows the equation system to be broken into two independent equation systems (decoupled Newton-Raphson method).

3.1. Modified Iterative Procedure for Decoupled Newton-Raphson Method

Standard methods for three-phase load flow calculation are based on the fact that all the phase powers are constant, which is not always true. With the connection of the traction substations onto the power system we are facing the problem of loads with two- phase constant power and we have to modify our standard calculation methods [6]. The calculation itself is done in a few steps:

STEP 1 - Declaration of the starting values for the calculation

For nodes in the network, it is necessary to declare the starting voltage magnitudes and angles. Usual values are: 1 p.u. for magnitudes, and 0° for angles. It should be kept in mind that the angles at phases are rotated by 120°, and that transformers cause rotation of the voltage angles, depending on the type of connection. The value of the internal busbars' voltage angle at the reference node is usually declared as 0.

STEP 2 - Standard methods' modification

In standard methods, loads are given by the constant per phase power, which is then compared with calculated powers within each step of the iteration. In order to include the two-phase load into the standard calculation, per phase power has to be found from the given two-phase power of the load using the expressions (9)-(11). Unlike the standard loads, phase powers are changing together with phase voltages. The derivates of the loads with regard to voltages are taken into the Jacobian matrix.

STEP 3 - Evaluation of the new voltage angles values

Using the expression (15) the active powers at the phases of all load nodes and terminal busbars of the generator node can be calculated, and then the difference between the calculated and the given powers (ΔP_i^p) has to be found. The active power, going into the network, for the internal busbars of the generator nodes (except reference) is calculated using the (17) and the difference between the calculated and given powers is $\Delta Pgen_j$. If the obtained values satisfy the given precision limits, then the current values for the voltage angles satisfy the given active power precision limits. Iterative procedure isn't, however, finished because the voltages have to satisfy the conditions stated in the STEP 4.

If the requested precision isn't fulfilled, then the iterative procedure is continued by declaring the new voltage angle values and going onto the next STEP.

STEP 4 - Evaluation of the new voltage magnitudes

Using the exp. (16) the reactive powers at the phases of all load nodes and terminal busbars of the generator node can be calculated, and the difference between the calculated and the given powers (ΔQ_i^p) has to be found. The difference between the phase *a* regulated voltages on the generator node terminal busbars is determined by the definition (18), and is equal to the difference between the real and the given voltage, respectively. Again these values are compared with the requested precision and if they fall within the approved interval, the current voltage magnitudes satisfy the requested precision of the reactive powers and regulated voltages.

In case all the conditions in both steps are fulfilled, the iterative procedure is completed, and the current values are the equation solutions and we go onto the next, STEP 5. If they aren't, we are returning onto the STEP 2.

STEP 5 - Evaluation of the load flows

When all the voltage values in the three-phase network nodes are found, the iterative procedure is completed and the calculation of branch

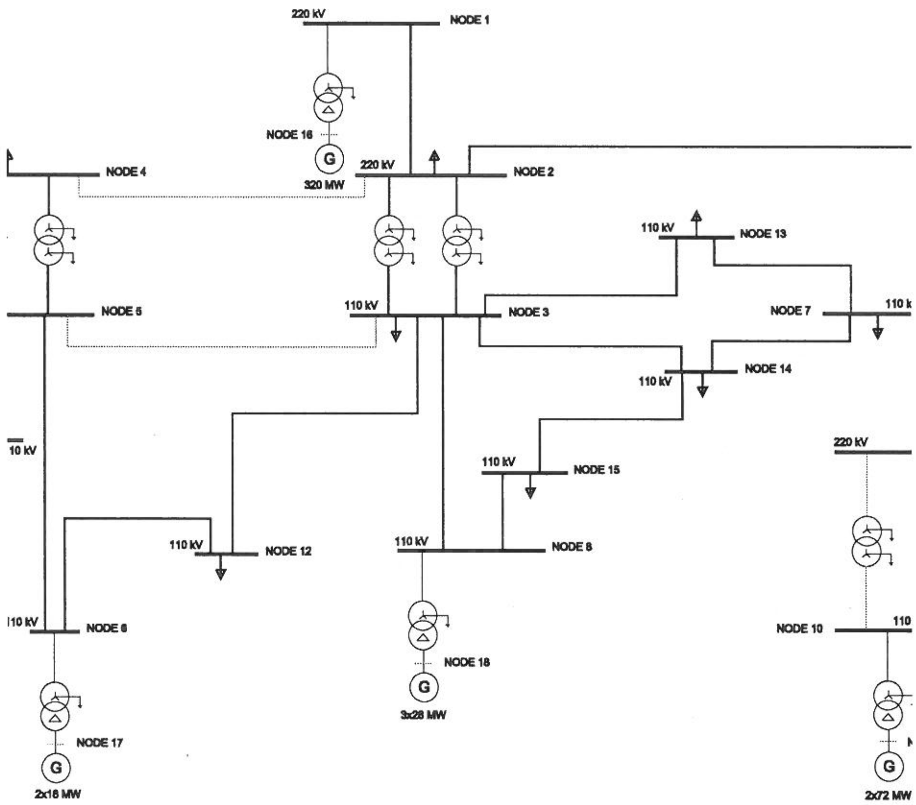

Fig. 2. Test Network.

Node	TYPE	U_{nom} (kV)	3-phase load (MW)	(MVAr)	2-phase load (MW)	(MVAr)	Generat. (MW)	Terminal voltage (p.u.)
1	PQ	220.0	.0	.0	.0	.0	.0	.000
2	PQ	220.0	-150.0	-50.0	.0	.0	.0	.000
3	PQ	110.0	-85.0	-35.0	.0	.0	.0	.000
4	PQ	220.0	-40.0	-25.0	.0	.0	.0	.000
5	PQ	110.0	-95.0	-40.0	.0	.0	.0	.000
6	PQ	110.0	.0	.0	.0	.0	.0	.000
7	PQ	110.0	-55.0	-25.0	.0	.0	.0	.000
8	PQ	110.0	.0	.0	.0	.0	.0	.000
9	PQ	220.0	.0	.0	.0	.0	.0	.000
10	PQ	110.0	.0	.0	.0	.0	.0	.000
11	PQ	110.0	.0	.0	-12.0	-9.0	.0	.000
12	PQ	110.0	.0	.0	-15.0	-10.0	.0	.000
13	PQ	110.0	.0	.0	-6.0	-2.0	.0	.000
14	PQ	110.0	.0	.0	-6.0	-3.0	.0	.000
15	PQ	110.0	.0	.0	-8.0	-5.0	.0	.000
16	PV	20.0	.0	.0	.0	.0	200.0	1.040
17	PV	10.5	.0	.0	.0	.0	35.0	1.030
18	PV	10.5	.0	.0	.0	.0	80.0	1.010
19	PV	10.5	.0	.0	.0	.0	70.0	1.050
20	SLACK	10.5	.0	.0	.0	.0	.0	1.040

Table 1. Load and Generator Data.

currents and load flows may begin according to the following expressions:

$$[I_{i-j}^{abc}] = [Y_u] \cdot ([V_i^{abc}] - [V_j^{abc}]) + [Y_p'] \cdot [V_i^{abc}] \tag{20}$$

$$[I_{j-i}^{abc}] = [Y_u] \cdot ([V_j^{abc}] - [V_i^{abc}]) + [Y_p'] \cdot [V_j^{abc}] \tag{21}$$

4. Numerical Examples

There is a PC program that implements the described algorithm. Part of Croatia's power system is used as a test network. In this network there are a few traction substations in a relative small area (Figure 2). Data considering the

iteration step	CASE 1		CASE 2	
	max P (p.u.)	max Q (p.u.)	max P (p.u.)	max Q (p.u.)
1	-.581436	-.186279	-.497874	.068384
2	.099037	-.082430	.084052	.031822
3	-.055250	.042848	-.017492	.014274
4	-.028725	.022741	-.009203	.007654
5	-.014914	.011961	-.004968	.004070
6	-.007910	.006308	.002670	.002108
7	-.004099	.003297	.001471	.001107
8	-.002273	.001738	-.000751	.000669
9	-.001135	.000905	.000455	.000334

Table 2. Maximal Unbalanced Power after Each Iteration Step for Both Cases.

loads and the given production is presented in Table 1.

Two situations have been analysed; CASE 1 where all the traction substations were connected at the same phases, while in CASE 2 the phases were changed in the circular motion. Convergence behaviour is presented in Table 2. The results for the current and voltage unbalances at all nodes are listed in Table 3. Examples of line currents and power flows are given in Table 4a for CASE 1 and in Table 4b for CASE 2.

5. Conclusions

Two-phase load modelling in three phase load flow by Newton-Raphson method is a useful tool when dealing with planning and analysis of the Power system, with the Electrical traction system - AC 25 kV, 50 Hz connected onto it. The software package is used for generator protection devices and high voltage motors analysis and adaptation. Negative current component in the system causes excess loses in generators and motors. The advantages and the benefits of this method are shown in a numerical example in the article. The analysis of the connection of the Traction substations onto the Power system can be performed based on this program, and the best possible solution can be identified.

Acknowledgements

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	CASE 1				CASE 2			
	V_{pos} (p.u.)	V_{neg} (p.u.)	I_{neg} (%)	V_{zero} (p.u.)	V_{pos} (p.u.)	V_{neg} (p.u.)	I_{neg} (%)	V_{zero} (p.u.)
1	1.0171	0.0283		0.0000	0.9989	0.0093		0.0000
2	1.0143	0.0288		0.0000	0.9960	0.0095		0.0000
3	0.9918	0.0396		0.0000	0.9714	0.0130		0.0000
4	1.0067	0.0307		0.0000	0.9880	0.0101		0.0000
5	0.9909	0.0399		0.0000	0.9707	0.0132		0.0000
6	0.9939	0.0406		0.0000	0.9734	0.0141		0.0000
7	0.9846	0.0412		0.0000	0.9640	0.0127		0.0000
8	1.0028	0.0383		0.0000	0.9816	0.0130		0.0000
9	1.0362	0.0230		0.0000	1.0204	0.0076		0.0000
10	1.0405	0.0157		0.0000	1.0286	0.0051		0.0000
11	0.9891	0.0424		0.0000	0.9689	0.0139		0.0000
12	0.9917	0.0425		0.0000	0.9713	0.0156		0.0000
13	0.9870	0.0411		0.0000	0.9665	0.0123		0.0000
14	0.9888	0.0414		0.0000	0.9683	0.0134		0.0000
15	0.9941	0.0416		0.0000	0.9733	0.0148		0.0000
16	1.0536	0.0212	7.1	0.0000	1.0365	0.0070	2.3	0.0000
17	1.0487	0.0303	12.1	0.0000	1.0241	0.0106	4.2	0.0000
18	1.0275	0.0292	12.2	0.0000	1.0040	0.0099	4.1	0.0000
19	1.0612	0.0165	6.6	0.0000	1.0476	0.0054	2.2	0.0000
20	1.0477	0.0106	4.6	0.0000	1.0386	0.0035	1.5	0.0000

Table 3. Table of Voltages and Negative Sequence Currents.

I	J	seq.	Current magn. (A)	Current angle (°)	Phase	Current real (A)	Current imag (A)	Power P (MW)	Power Q (MVAr)
		Pos	622	104.1	A	-217	626	76.86	39.82
1	2	Neg	70	160.6	B	611	-239	63.52	52.49
		Zero	0	0.0	C	-394	-387	59.62	41.29
		Pos	625	-76.4	A	213	-631	-76.81	-40.27
2	1	Neg	70	-19.4	B	-613	245	-63.47	-52.88
		Zero	0	0.0	C	400	386	-59.58	-41.86
		Pos	336	112.0	A	-172	327	45.65	15.34
2	4	Neg	49	160.9	B	342	-95	37.03	24.48
		Zero	0	0.0	C	-170	-233	34.17	16.05
		Pos	340	-69.4	A	165	-334	-45.56	-15.86
4	2	Neg	49	-19.1	B	-344	103	-36.94	-24.95
		Zero	0	0.0	C	179	231	-34.11	-16.85

Table 4a. Example of Line Currents and Power Flows for CASE 1.

I	J	seq.	Current magn. (A)	Current angle (°)	Phase	Current real (A)	Current imag (A)	Power P (MW)	Power Q (MVAr)
		Pos	635	103.4	A	-162	600	67.11	42.85
1	2	Neg	23	230.6	B	632	-185	69.36	45.90
		Zero	0	0.0	C	-470	-415	63.53	47.20
		Pos	639	-77.0	A	158	-605	-67.07	-43.31
2	1	Neg	23	50.6	B	-633	191	-69.31	-46.30
		Zero	0	0.0	C	476	414	-63.48	-47.63
		Pos	345	110.6	A	-134	312	39.50	17.80
2	4	Neg	17	223.1	B	357	-61	40.71	20.46
		Zero	0	0.0	C	-223	-251	36.72	20.81
		Pos	349	-70.7	A	128	-318	-39.41	-18.38
4	2	Neg	17	43.1	B	-359	70	-40.62	-20.93
		Zero	0	0.0	C	232	249	-36.64	-21.37

Table 4b. Example of Line Currents and Power Flows for CASE 2.

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